A Level Maths Induction Work

The A level Maths transition work is set on an online learning platform called Integral. This is an online resource that you will use a lot during your A level Maths studies. The videos, activities and assessments in this work will review key ideas from GCSE Maths whilst also starting to look at things from an A level Maths viewpoint.

We are asking you to complete the **Surds & Indices** and **Algebraic Manipulation** sections, but we highly recommend that you work through some of the other sections as well to give yourself the best preparation for A level Maths. As part of the induction process there will be a short test in one of your maths lessons in the first week of term, your teachers will confirm the exact date on the enrolment day in September. This test is based on key skills from GCSE Maths. A practice test is included at the end of this document – the real test is very similar.

You should bring your marked practice test to your induction lesson on the enrolment day in September. Your teachers can offer support on any aspects you have struggled with ahead of the actual test the following week.

We expect students to get at least 70% in the induction test (80% for those doing Further Maths).

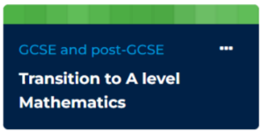
Accessing the resources

To access the resources, go to https://integralmaths.org/.

Your login will be 897T- followed by your first initial and surname. For example, someone called Sam Jones would have the login 897T-sjones. Your password is initially the same as your username.

Accounts have been created based on the names from the July induction day. If your account does not exist yet please email Mr Chapman on dqc@hardenhuish.wilts.sch.uk to have one created.

Note - if your surname has a space or hyphen in it, please omit this – for example Alex Rain-Wind would become 897T-arainwind.



Once logged in select "Transition to A level Mathematics".

You will then see a welcome message, and an introduction video which you need to watch before the materials become available.

There are seven sections of the course, but we are only asking you to complete **Surds & Indices** and **Algebraic Manipulation** sections.

Please work through the interactive videos before attempting the assessment. If you wish you can explore the "Going Deeper" materials or work through some of the other sections.

Mathematical Enrichment

The summer between Year 11 and Year 12 is a really good opportunity for you to explore your interest in your A level options. There is so much excellent media to inspire you and get you excited about studying mathematics at A Level. Here are some options of what you could do :

Books

Here are some of the maths department's favourites:

Fermat's Last Theorem by Simon Singh	**Unashamed** by Harry Baker	**The GCHQ Puzzle Book**
Chaos by James Gleick	**Why do buses come in threes?** By Rob Eastaway	**Physics Impossible** by Michio Kaku
The music of the primes by Marcus du Sautoy	**Hello World** By Hannah Fry	**Humble Pi** By Matt Parker
17 equations that changed the world by Ian Stewart	**The Simpson's and their mathematical secrets** By Simon Singh	**Alex's Adventures in Numberland** By Alex Bellos
Archimedes' Revenge By Paul Hoffman	**The art of statistics** By David Spiegelhalter	**How to make the world add up** By Tim Harford

Podcasts

While not all strictly about maths, these podcasts are excellent examples of applications of mathematics in the world:

More or Less: Behind the stats – BBC Radio 4

The Naked Scientists podcast – The Naked Scientists

The Curious Cases of Rutherford & Fry – BBC Radio 4

The Numberphile Podcast – Numberphile

Youtube Channels

These channels are all brilliant and engaging demonstrations of mathematics and its applications.

- Numberphile - https://www.youtube.com/@numberphile
- Veritasium - https://www.youtube.com/@veritasium
- 3blue1brown - https://www.youtube.com/@3blue1brown
- Steve Mould - https://www.youtube.com/@SteveMould
- Tom Scott - https://www.youtube.com/@TomScottGo

Documentaries

BBC Horizon on Fermat's Last Theorem – this tells a truly amazing story of the British Mathematician Andrew Wiles and his journey to find the proof for a problem that mathematicians have been working on since the 17th century.

BBC iPlayer - Horizon - 1995-1996: Fermats Last Theorem

Practice Test

Your test will ask similar questions to this one. Answers are on the following page.

You may NOT use a calculator

1. Expand and simplify

(a) $4x(3x - 2) - x(2x + 5)$ (b) $(2x + 3)(2x - 1)$ (c) $(a - 12)^2$

2. Factorise

(a) $x^2 - 7x$ (b) $x^2 + 9x - 36$ (c) $y^2 - 64$ (d) $25y^3 - 9y$

3. Simplify

(a) $\frac{4x^3y}{8x^2y^3}$ (b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

4.Solve the following equations

(a) $\frac{h-1}{4} + \frac{3h}{5} = 4$ (b) $x^2 - 8x = 0$

(c) $k^2 - 7k - 18 = 0$ (d) $p^2 + 4p = 12$

5. Write each of the following as single powers of x and/or y

(a) $\frac{1}{x^4}$ (b) $(x^3y)^4$ (c) $\frac{x^5}{x^{-2}}$

6. Evaluate, leaving your answer as a fraction where appropriate.

(a) 4^{-2} (b) 10^0 (c) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

7. Solve the simultaneous equations

$$3x - 5y = -11$$

$$5x - 2y = 7$$

8. Rearrange the following equations to make x the subject

(a) $v^2 = u^2 + 2ax$ b) $V = \frac{1}{3}\pi x^2 h$ (c) $y = \frac{x+2}{x+1}$

9. Solve $x^2 + 4x + 1 = 0$, giving your solutions in exact form.

10. Solve $5x^2 - x - 1 = 0$, giving your solutions in exact form.

Solutions to practice test

1. Expand and simplify

(a) $4x(3x - 2) - x(2x + 5)$

$= 12x^2 - 8x - 2x^2 - 5x$

$= 10x^2 - 13x$

Common errors:

- Sign errors on the second bracket -- often students get $+5x$ rather than $-5x$.

(b) $(2x + 3)(2x - 1)$

$= 4x^2 - 2x + 6x - 3$

$= 4x^2 + 4x - 3$

Common errors:

- Simplifying $-2x + 6x$ to $-8x$. Remember you start with $-2x$ and add $6x$ onto it so you end up with a positive value, rather than going further into the negatives.

(c) $(a - 12)^2$

$= (a - 12)(a - 12)$

$= a^2 - 24a + 144$

Common errors:

- Just squaring both terms and achieving $a^2 - 144$. Remember this represents the entire bracket multiplied by itself so it is an expanding double brackets question.

2. Factorise

(a) $x^2 - 7x$

$= x(x-7)$

Common errors:

- Students often forget how to simplify something simple like this because they expect a quadratic to have 3 terms and therefore factorise into double brackets. Remember to look for the opportunity to just pull out a common factor between two terms.

(b) $x^2 + 9x - 36$

Identify a factor pair of -36 that sum to $+9$

$= (x+12)(x-3)$

Common errors:

- Not considering all of the factor pairs and settling on the first factor pair you think of e.g. 9 and 4—remember the numbers need to sum to give the coefficient of x and multiply to give the constant term.

(c) $y^2 - 64$

$= (y+8)(y-8)$

Common errors:

- Failing to recognise the form of a difference of two squares! Anything of the form $a^2 - b^2$ factorises into $(a+b)(a-b)$

(d) $25y^3 - 9y$

$= y(25y^2 - 9)$

$= y(5y+3)(5y-3)$

Common errors:

- Not spotting that both terms have a factor of y in common. By factorising this out first, you end up with a difference of two squares problem. Remember to look for ways to make the problem simpler.

3. **Simplify**

(a) $\frac{4x^3y}{8x^2y^3}$

$= \frac{x}{2y^2}$

Common errors:

- If you try to do everything in a rush you can miss factors and make basic arithmetic mistakes. Take these questions slow, consider numerical factors, then algebraic factors letter by letter and ensure you have been systematic in your approach.

(b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

$= \frac{2(3x+2)}{6} + \frac{4x-1}{6}$

$= \frac{6x+4+4x-1}{6}$

$= \frac{10x+3}{6}$

Common errors:

- Sometimes people multiply the denominators together to get a common denominator of 18. While this works, you will end up having to do simplification later on where you may make an error. Remember you only need to find the lowest common multiple, in this case: 6.
- People tend to find a common denominator without a problem but make errors multiplying the numerators.

4.Solve the following equations

(a) $\frac{h-1}{4} + \frac{3h}{5} = 4$

Multiply through by both denominators

$5(h-1) + 4(3h) = 4 \times 4 \times 5$

Expand and simplify

$5h - 5 + 12h = 80$

$17h = 85$

$h = 5$

Common errors:

- Neglecting to multiply the first numerator by 5 and the second numerator by 4.

(b) $x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0$ or $x = 8$

Common errors:

- Forgetting the basics of factorising! People often think you need to use a factorising method that finds the factor of the constant that sum to give the coefficient of x then become stuck rather quickly. Remember both of these terms have a common factor of x so can factor it out.
- Dividing through by x. This loses $x = 0$ as a solution.

(c) $k^2 - 7k - 18 = 0$

Factors of -18 that sum to -7 : $-9\ and + 2$

$(k - 9)(k + 2) = 0$

$k = 9$ and $k = -2$

Common errors:

- Sign errors, giving $k = -9$ and $k = 2$ as the solutions. Remember you are trying to make the brackets $= 0$.

(d) $p^2\ +\ 4p\ =\ 12$

$p^2 + 4p - 12 = 0$

$(p + 6)(p - 2) = 0$

$p = -6$ or $p = 2$

Common errors:

- Guessing instead of rearranging the equation to set it $= 0$. Start by subtracting 12 from both sides to get a more familiar three term quadratic equation to solve.

5. Write each of the following as single powers of x and/or y

(a) $\frac{1}{x^4}$ $= x^{-4}$

(b) $(x^3y)^4 = x^{12}y^4$

(c) $\frac{x^5}{x^{-2}} = x^{5--2} = x^7$

Common errors:

- Forgetting your laws of indices! Remember the basics:
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{m\times n}$
- Not recognising the index applies to **everything** in the bracket in b.
- Double negative errors on c.

6. Evaluate, leaving your answer as a fraction where appropriate.

(a) 4^{-2}

$= \left(\frac{1}{4}\right)^2$

$= \frac{1}{16}$

(b) 10^0

$= 1$

(c) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{8}{27}}$

$= \frac{2}{3}$

Common errors:

- Forgetting the next set of laws of indices, remember:
  - $a^{-m} = \frac{1}{a^m}$ (people often <u>wrongly</u> think this is $a^{-m} = -a^m$)
  - $a^0 = 1$ <u>anything to the power of zero is 1!!!</u>
  - $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

7. Solve the simultaneous equations

(1) $3x - 5y = -11$

(2) $5x - 2y = 7$

$5 \times (1) : 15x - 25y = -55$

$-\ 3 \times (2) : 15x - 6y = 21$

$-19y = -76$

$y = 4$

Let $y = 4$ in (2)

$5x - 2(4) = 7$

$5x - 8 = 7$

$5x = 15$

$x = 3$

$$\therefore \boldsymbol{x = 3\ and\ y = 4}$$

Common errors:

- Forgetting to multiply the right hand side of the equation
- Sign errors very common here
- Don't stop when you find one solution, remember you are trying to find a **pair** of values!

8. Rearrange the following equations to make x the subject

(a) $v^2 = u^2 + 2ax$

$$2ax = v^2 - u^2$$

$$x = \frac{v^2 - u^2}{2a}$$

b) $V = \frac{1}{3}\pi x^2 h$

$$3V = \pi x^2 h$$

$$x^2 = \frac{3V}{\pi h}$$

$$x = \pm\sqrt{\frac{3V}{\pi h}}$$

(c) $y = \frac{x+2}{x+1}$

$$y(x+1) = x+2$$

$$yx + y = x + 2$$

$$yx - x = 2 - y$$

$$x(y-1) = 2 - y$$

$$x = \frac{2-y}{y-1}$$

Common errors:

- Forgetting the $\pm$ when square rooting on b.
- Not knowing how to deal with x appearing in two places on c. The objective is to get every x on the LHS of the equation then to <u>factorise</u> to get it appearing in just one place.

9. Solve $x^2 + 4x + 1 = 0$, giving your solutions in exact form.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 4}}{2}$$

$$x = \frac{-4 \pm \sqrt{12}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$x = -2 \pm \sqrt{3}$$

10. Solve $5x^2 - x - 1 = 0$, giving your solutions in exact form.

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(5)(-1)}}{2(5)}$$

$$x = \frac{1 \pm \sqrt{1 + 20}}{10}$$

$$x = \frac{1 + \sqrt{21}}{10}$$

Common errors for 9 and 10:

- Missing the $\pm$ in the quadratic formula (or otherwise misquoting the formula
- Skipping steps. Make sure you write down the initial substitution into the formula and show the simplification of the expression under the square root. You are less likely to make mistakes if you take these steps slowly.
- Missing the simplification of the surd in 9
- Sign errors. More likely in 10.