What a load of old units!

By now you will have realised (by reading the colour supplements or possibly even by listening to your Science teachers!) that we have been using a system of unit commonly referred to as the S.I. System. (Systeme Internationale for French speakers amongst you). However you will also have noticed that these are by no means the only units in use, pounds(lb.)) and stones belong to a system called Imperial, as do feet and inches. Energy could be measured in calories or even ergs, the latter of which belongs to a system called c.g.s.! - old science books often contain units from this system. Why do so many units exist and how can different units be used for the same thing? To answer this we need t o know something about 'Physical quantities'!

Physical Quantities These are the 'words' that the 'language' of Physics uses to talk about the universe. You are familiar with many already - force, mass, area, pressure, acceleration etc. are all physical quantities, they have a numerical size and a unit

e.g. the mass of a blue whale = 6 000 kg
size unit

Both are vital if the language is to communicate anything useful - if in doubt think how much this statement is

the mass of a blue whale = 6 000 ....!

New physical quantities are defined using equation s containing more basic ones.

For example, Pressure is defined by the equation:

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The pressure, P, acting on a surface is equal to the force, F, on that surface divided by its area, A. The equation also gives us a unit for pressure.

Using S.I. units

$$P = \frac{\text{force in newtons}}{\text{area in metres}^2}$$

Thus the units of pressure are newtons/metres^2 (N/m^2 or Nm^{-2})

It is often convenient to call this 'combination of units' something else, for example the Pascal, (Pa) is defined so that 1 Pa is a pressure of 1 N/m^2 and we have a new S.I. unit, the Pa.

Had we measured the force in dynes (c.g.s.) and the area in cm^2 our unit for pressure would be the dyne/cm^2. Get the picture? A system of units is built up using equations which define the new physical quantities to give the new units as well.

Which comes first, the chicken or the egg?

Because all physical quantities are defined based on others there has to be a starting point. So some physical quantities are defined not to depend on any others and are defined in a rather offhand, arbitrary way. These quantities are determined base quantities (or *dimensions*) and their units base units. There are seven of these, but only six are in common use in A level syllabuses.

Quantity(dimension)	Symbol	S.I. Unit	Unit Symbol
length	L	metre	m
mass	M	kilogram	kg
time	T	second	s
electric current	I	ampere	A
thermodynamic temp	Θ	kelvin	K
amount of substance	N	mole	Mol

These units are often defined in intriguing ways and in a special order since some of them depend on others. For example: The definition of a metre used to run like this 'One metre is the length equal to 1 650 763.73 wavelengths in a vacuum of the radiation (light) corresponding to the transition between the levels $2p_{10}$ - $5d_5$ of the krypton-86 atom'. and the kilogram 'One kilogram is equal to the mass of the international prototype kilogram'. (a cylinder of platinum-iridium kept at the Bureau Internationale des Poids et Mesures (BIPM), Sevres, Paris. All other 1kg masses have to be compared to this one.

So what??

All very interesting I hear you say but how is this going to get me an A level in Physics??? All of the more complex units like newtons, hertz, etc. are built up from the basic six above and are called derived units. to get from basic units to derived units you need to know the formula liking the physical quantities. You can also use this to check that formulae are 'dimensionally correct' and the units of constants in formulae.

Example: Could the equation $v = f\lambda$ correctly describe the relationship between v, f and λ for a wave?

We must break each of the quantities down into its base quantities which can be represented by the dimensions M, L, T etc. We usually write dimensions in square brackets so as not to get ourselves confused between M the mass dimension and m the length unit!!

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \frac{\text{frequency (number of vibrations)}}{\text{time}} \times \text{length}$$

$$= \frac{[L]}{[T]} = \frac{\text{a number}}{[T]} \times [L]$$

SO $\frac{[L]}{[T]} = \frac{[L]}{[T]}$ OK sigh of relief!

This equation is dimensionally correct, just as well since we've been using for the last few years!. All equations have to be dimensionally correct - it would be nonsense to say that mass x length = time for example!

However it is possible that the correct relationship between velocity, frequency and wavelength is something like $v = 6 f\lambda$, because this method of looking at equations cannot deal with numbers (constants) that have no units (dimensions).

What about the constants?

In some equations, like the example above, there is no constant or rather the constant is equal to 1. However this is not always true. For example let us look at the stretching of a spring. The formula which links the extension of the spring to the force applied to it is called the Hooke law.

$$F = k\,x$$

force = constant x extension

since F = ma (Newton's second law, you remember!) we have:-

ma = constant x extension

Putting in the dimensions we have:

$$\frac{[M]\,[L]}{[T^2]} = \text{constant} \times [L]$$

On the face of it this looks dimensionally incorrect - not allowed - but what if the constant itself has dimensions.

$$\frac{[M]\,[L]}{[T^2]} = ?? \times [L]$$

$$\frac{[M]\,[L]}{[T^2]} = \frac{[M]}{[T^2]} \times [L]$$

So the constant, k, in the Hooke law must have dimensions $\frac{[M]}{[T^2]}$.

Rearranging the equation we can see that k = F/x = force/extension = newtons/metre. Check that the dimensions of newton/metre are $[M]/[T^2]$

BINGO !! AND WELL DONE MR HOOKE!

More Questions for you to do!

1 What is the mass of a blue whale in SI units (approx)?

2 Which formula defines the new physical quantity force in newtons?

3 Use the method of dimensions to show that the formula $F = \frac{G M_1 M_2}{r^2}$

is dimensionally correct when F = gravitational force, M_1 and M_2 are masses, r is the distance between the masses and G is a constant whose units are Nm^2kg^{-2}.

4 Amazingly pressure $P = h\rho g$ where h is depth of fluid, g is gravitational field strength, and ρ is the density (mass per unit volume)

Rearrange the equation, put in the units, and find out the units of g.
g can also be measured in ms^{-2}. Use the method of dimensions to check that the equation is still correct.

5 You may be surprised to note that the joule is not a base unit. There are several equations which could be used to define the physical quantity energy. You should be able to find at least two of them.

6 Use one of your equations from question 5 to find the combination of base units equivalent to the joule.

Initiative exercises!

7 Use the library or other sources to find the definitions of the six base quantities in the SI system.

8 Give the new definition of the metre and try to explain why it is better than the old one.

9 Name one problem with using a prototype metre as a standard of length.

10 One of the base quantities was omitted from the list - what is it and what is its definition?